Quantitative determination of taper adequate to provide resistance form: Concept of limiting taper

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The dentist must ensure that the taper resulting from controlled tooth preparation is adequate to provide required retention and resistance. As a dentist prepares a tooth, the angulation and taper of the bur establishes the taper at each point on the tooth. For some surfaces the taper may be constant along the axial wall of the preparation. Frequently, however, it is necessary to vary the taper at different locations on the preparation, as required in "two-plane" reduction. The taper of the preparation, with all of its variations, determines the quality of retention and resistance of the preparation. Usual recommended tapers of 2 to 5 degrees are based on studies that have related taper to retention, not to resistance form.[1-5] This is true even though it has been stated that resistance form is a more general concept.[6] The existing void in emphasis and knowledge relating adequate taper to resistance was addressed by Weed and Baez.[7] They presented a method for determining the taper of a preparation that would be adequate to provide resistance form as based on the diameter and length of the preparation. The relevance of this relationship used in determining existing resistance is of great clinical importance.

A study was conducted to mathematically address the relationship of taper to resistance to provide quantitative clinical guidelines for determination of taper adequate to provide resistance form. Specific clinical stituations are discussed to demonstrate the mathematical principles.

RESISTANCE FORM

Clinical evaluation

Resistance form of a preparation is provided if the side of the preparation interferes with the arc of the casting pivoting about a point on the opposite side of the preparation.[3] A preparation can have resistance to rotation in one direction but not in another. To be clinically acceptable the preparation must resist rotation in all directions, buccal, lingual, mesial, and distal. The uncemented casting, or the wax pattern, can be tested for resistance on the die by attempting to roll it off of the die in each direction. If opposition to dislodgement is met in all directions, the preparation can be considered to have resistance form.

Direction of arc

With a two-dimensional model[8] resistance can be evaluated by determining the direction of arc for each point of the rotating restoration (Fig. 1). The center of rotation is the margin on the opposite side of the restoration. A radius is drawn from this center of rotation to the point of the restoration to be evaluated. The tip of the rotating radius scribes an arc that shows the path followed by the restoration at that point. In Fig. 1 six points on the restoration are evaluated. Note that the direction of each arc is different. The direction of arc depends on the location on the preparation and changes accordingly. If a large enough arc is scribed it can be seen from the curvature that the direction changes continuously. The direction of interest is in the first instant of rotation. This direction is specified by a line that is tangent to the arc at that point. This line is also perpendicular to the radius at the point for which resistance form is being evaluated (Fig. 2). If the direction of arc is into the tooth, that point has resistance. If the direction of arc does not penetrate the tooth, it does not have resistance. For a restoration to have resistance to rotation in a given direction there must be at least one point that is resistive in that direction. If the restoration is considered to be a rigid body, resistance of even one point will prevent rotation of the restoration.

"On-off" nature

In evaluating resistance at different points, a unique property of resistance becomes evident; that is, for each point there either is resistance or there is not. Like a light switch, it is either on or off. For points 1, 2, and 3 in Fig. 1 the direction of arc is into the tooth so the resistance is "on." For points 4, 5, and 6 the direction of arc is away from the tooth so that resistance is "off." A geometric guideline to determine whether the resistance will be on

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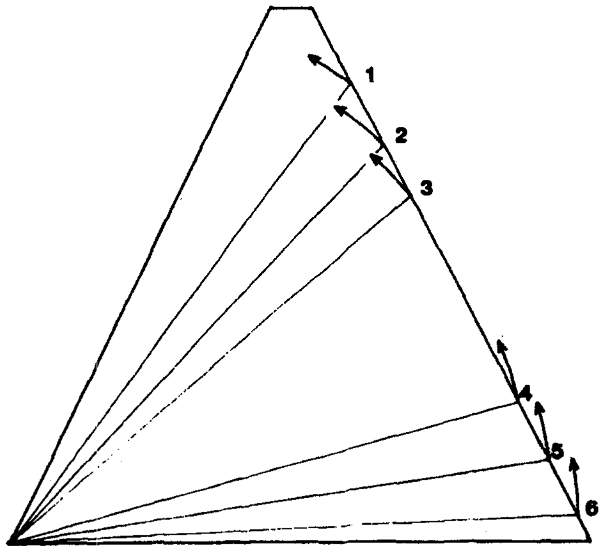

Fig. 1. Resistance form at each of six points is evaluated by determining direction of arc. Points *1, 2,* and *3* are resistive because direction of arc is into tooth. Points *4, 5,* and *6* are not resistive because direction of arc is away from tooth.

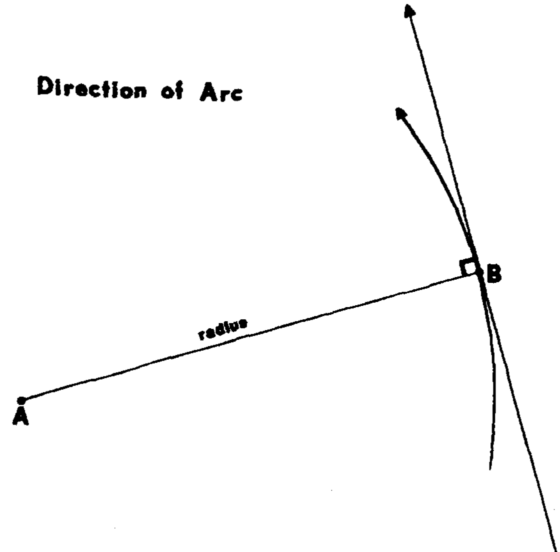

Fig. 2. Direction of movement of arc at point *B* is given by line tangent to arc at that point. This line is also perpendicular to radius.

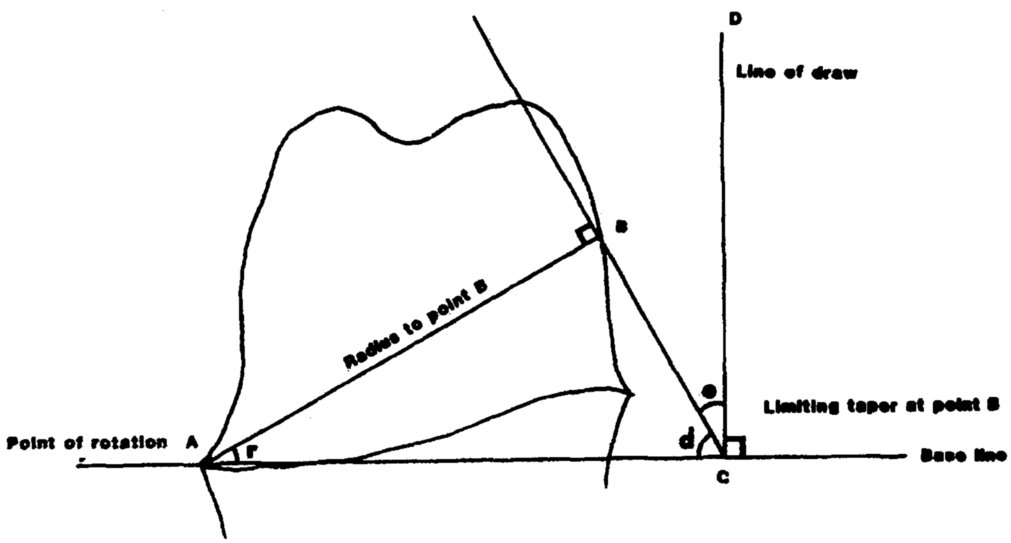

Fig. 3. Direction of arc at point *B* is specified by line *BC* which is tangent to arc and perpendicular to radius. Angle θ between this line and line of draw is defined as limiting taper. Equation 1 shows that $\theta = r$.

or off is found by comparing the slope of the preparation to the direction of arc. Note that for all points that are not resistive, the slope of the preparation is more shallow than the direction of arc. For all points that are resistive, the slope of the preparation is steeper than the direction of arc. If the direction of the arc is examined at a point, any slope of the preparation more shallow than this arc will not have resistance and any slope of the preparation that is steeper than the direction of arc will have resistance. Clearly the direction of arc at a point specifies a boundary such that tapers on one side of it provide resistance whereas tapers on the other side do not.

LIMITING TAPER

Clinically it would be useful to know for each point on a preparation at what taper the resistance turns from *off* to *on*. The direction of arc gives this boundary. The initial direction of the curved arc is given by a line perpendicular to the radius. The taper of this line specifies the direction of arc at the initial movement. We define the taper of this line as the limiting taper. The quantitative relationship determines that at some point on a preparation any taper less than the limiting taper will provide resistance form, and any taper greater than the limiting taper will not provide resistance form. For

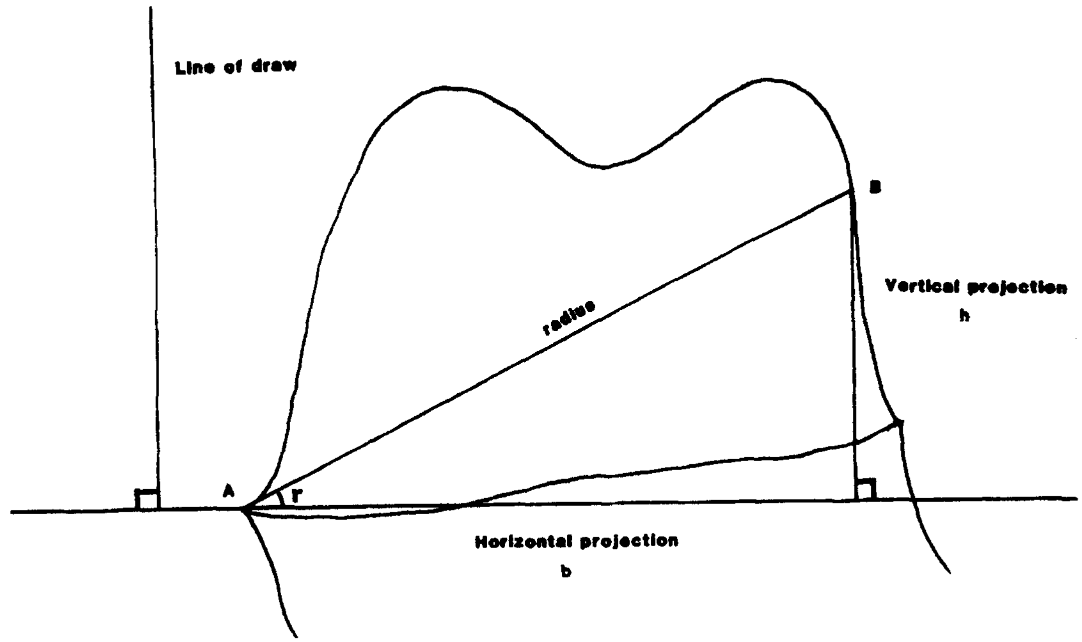

Fig. 4. Position of *B* is given by its vertical and horizontal projections, *h* and *b*. Equation 2, θ = Arctan h/b, expresses limiting taper as a function of position on preparation.

example, if the limiting taper at a point is 19 degrees, any taper between 0 and 19 degrees would provide resistance. The dentist has a range of tapers that would be adequate to provide resistance. On the other hand, if the limiting taper is 2 degrees, the taper must be less than 2 degrees to provide resistance. This is a more demanding situation and other surfaces should be considered to provide resistance. The limiting taper specifies an exact quantitative boundary for taper adequate to provide resistance. As with the direction of arc, the limiting taper varies as a function of location on the preparation. At different locations the limiting taper will be different. For surfaces on the preparation where the limiting taper is small the dentist is more limited in the range of taper that can provide resistance. For all preparations there are points and surfaces that will not be resistive. A preparation cannot be made for which all points are resistive. By using the concept of limiting taper the dentist can select those surfaces to provide resistance where the limiting taper is larger to facilitate and ensure clinical success. The mathematical solution for limiting taper presented in the following section provides the dentist with a means of determining the theoretical limiting taper for each point on the preparation.

Mathematical derivation

For an arbitrary preparation and line of draw (Fig. 3), a derivation of limiting taper as a function of position on the preparation will be presented. Point A is the center of rotation of the crown about which resistance form is being tested. The baseline AC is formed perpendicular to the line of draw and through point A, the center of rotation. The baseline for well-aligned teeth will usually fall near the preparation margin. For tilted abutments the baseline may not fall along the margin. This presents a warning sign to the dentist and will be discussed in the next section. The radius extends from point A to point B. Point B is the point at which the limiting taper is to be calculated. Angle r is the angle formed between the base line and radius. A line perpendicular to the radius at point B is extended to intersect the base at point C. As mentioned, this line specifies the initial direction of arc at point B and the taper of this line is defined as the limiting taper. The angle θ that this line BC makes with the line of draw is the limiting taper at point B. Since angles ACD and ABC are both 90 degrees it follows that

$$\begin{aligned} & r + d = 90^\circ && (1) \\ \text{and} \quad & \theta + d = 90^\circ \\ \text{which gives} \quad & \theta = r \end{aligned}$$

Equation 1 expresses the limiting taper θ as a function of its position on the preparation as given by r the angle made by the radius with the baseline. The angle θ can also be expressed in terms of h and b where h is the height of point B above the baseline and b is the horizontal projection of the radius on the baseline. From Fig. 4 we see that

$$\begin{aligned} & \tan r = h/b && (2) \\ \text{or} \quad & r = \arctan h/b \\ \text{and from eq. 1} \quad & \theta = \arctan h/b \end{aligned}$$

If the sides of the preparation are assumed to be straight with constant taper (Fig. 5), it is possible to determine the resistive and nonresistive regions. The point at which

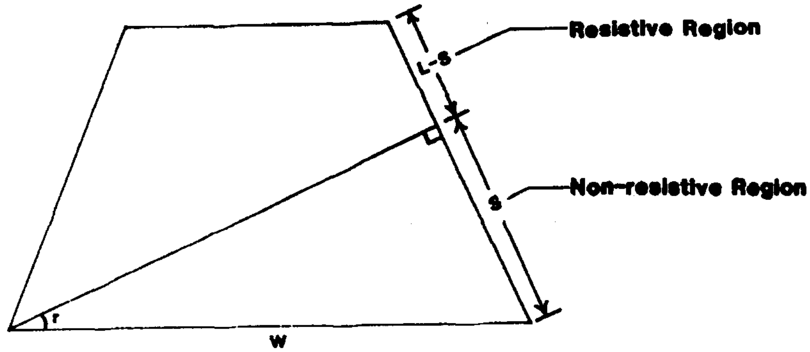

Fig. 5. For a preparation with straight walls, length of nonresistive region can be found. From equation 3, s = w × sin θ. Length of resistive region is (L − s) where L is length of side.

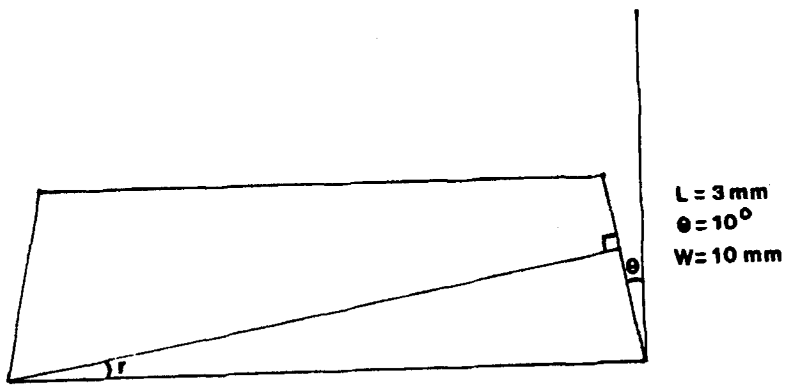

Fig. 6. Resistive and nonresistive regions of a preparation with a base of 10 mm, axial wall of 3 mm, and taper of 10 degrees are determined using equation 3. Nonresistive length s = 10 mm × sin 10 = 1.73 mm and resistive region is (3 mm − 1.73 mm) or 1.26 mm.

the radius is perpendicular to the side is the point at which the limiting taper equals the taper of the preparation. All points above this point are resistive and all points below it are not. This is true since the limiting taper equals r, the angle formed by the radius, which becomes larger for higher points and smaller for lower points. Since the taper remains constant, the limiting taper is greater than the taper at higher points and less than the taper at lower points. This determines the resistive and nonresistive regions. From Fig. 5,

$$\sin\theta = \sin r = s/w \qquad (3)$$
$$\text{or} \quad s = w \times \sin\theta$$

where s is the length of the nonresistive region, w is the length of the base, and r, the angle of the radius, is equal to the limiting taper and in this case the taper of the preparation. These equations can be manipulated for use in different forms dependent on the information desired.

CLINICAL APPLICATIONS

Resistive and nonresistive regions

The preparation in Fig. 6 can be evaluated to determine the resistive and nonresistive regions. The base is 10 mm, the side is 3 mm, and the taper is 10 degrees. From equation 3,

$$s = 10 \text{ mm} \times \sin 10$$
$$s = 1.73 \text{ mm}$$

The nonresistive region is 1.73 mm and the resistive region is 3 mm minus s or 1.27 mm. By these calculations the preparation is resistive, a conclusion that differs with the statement by Weed and Baez[7] in their Fig. 1 that the casting will dislodge.

Maximum resistive taper

Suppose in Fig. 6 that the taper could be varied. It would be useful to know exactly how tapered the preparation could be and still have resistance form. Since the highest point has the greatest limiting taper it would be the last to lose resistance. Thus the limiting taper of the highest point is also the limiting taper of the side. Using equation 3,

$$\sin\theta = L/B$$
$$\theta = \arcsin (3 \text{ mm}/10 \text{ mm})$$
$$\theta = 17.45\ldots$$

Thus the taper of the side can be any angle less than

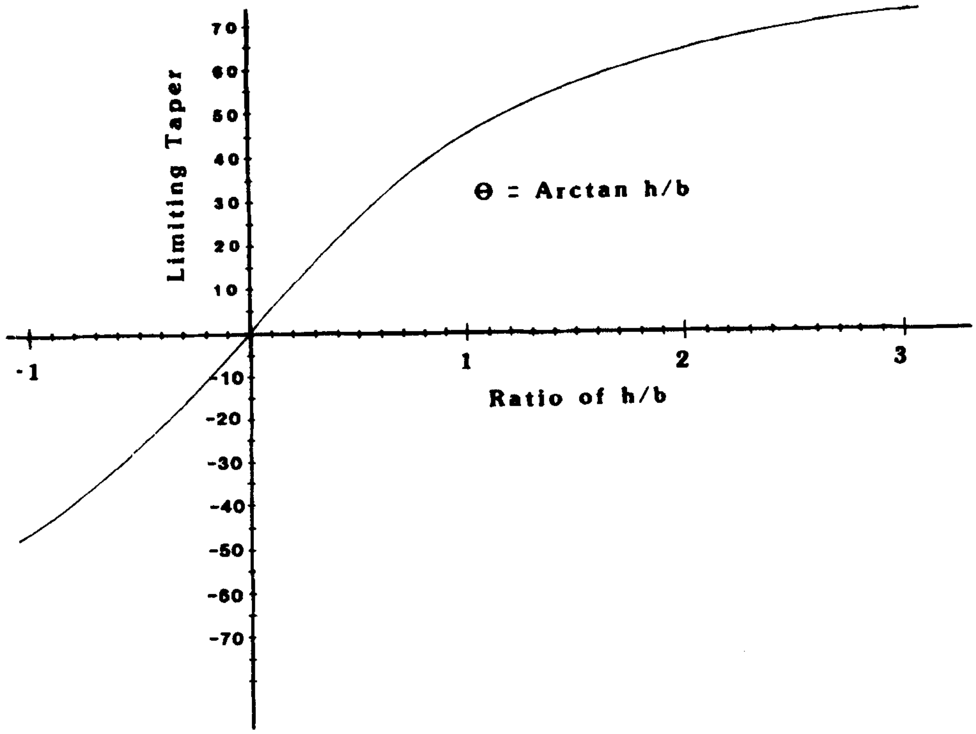

Fig. 7. From graph of equation 2, θ = Arctan h/b, quantitative values of limiting taper can be obtained.

17.45 degrees and still be resistive. Note that to use equation 3 the preparation is assumed to have flat sides and the base is perpendicular to the line of draw. For preparations with curved sides, equations 1 and 2 must be used.

Quantitative values of limiting taper

Limiting taper as determined in equation 2 is graphed in Fig. 7. Specific values are given in Table I. For example, a taper of 45 degrees is resistive for a ratio of h/b greater than 1. For anterior preparations where the ratio of height to base of some points is frequently greater than 1, the taper can be 45 degrees and still be resistive. Notice that a point with a height-to-base ratio of 0.1 can be resistive if the taper is less than 5.7 degrees. The standard recommended range of 2 to 5 degrees falls within this range. At first glance it would seem that the height-to-base ratio of even short molars would have points with ratios greater than 0.1 and that 2 to 5 degrees might always be a sure range of tapers to achieve resistance. Notice however that there are negative values for h/b and also for the limiting taper. This occurs when the baseline is not level with the preparation margin so that some points on the preparation are below the baseline. When this occurs the radius to any of these points forms an angle with the baseline that is negative. Similarly, the height of a point is measured from the baseline. If the point is below the baseline, the height is negative. For a point above the base line, the height is measured from the baseline and not the preparation margin. For the taper to be resistive it must be less than the limiting taper. With a negative limiting taper, tapers would have to be negative, or undercut, to be resistive. This is not feasible. Zero degrees is the lowest possible clinical taper without undercutting. Thus no resistance can be provided by any point that has a negative limiting taper. This important clinical situation is discussed in the next section.

Table I. Limiting tapers for values of h/b are derived by using equation 2, Θ = arctan h/b

h/b	Limiting taper	Resistive tapers	Nonresistive tapers
−0.5	−26.6	None	$t \geq 0$
−0.1	−5.7	None	$t \geq 0$
0.1	5.7	$0 \leq t < 5.7$	$t \geq 5.7$
0.2	11.3	$0 \leq t < 11.3$	$t \geq 11.3$
0.4	21.8	$0 \leq t < 21.8$	$t \geq 21.8$
0.7	34.9	$0 \leq t < 34.9$	$t \geq 34.9$
1.0	45.0	$0 \leq t < 45.0$	$t \geq 45.0$
1.5	56.3	$0 \leq t < 56.3$	$t \geq 56.3$
2.0	63.4	$0 \leq t < 63.4$	$t \geq 63.4$
3.0	71.6	$0 \leq t < 71.6$	$t \geq 71.6$

h = Vertical projection; b = horizontal projection.

Tilted abutments — beware

Fig. 8 demonstrates the idealized preparation of a tilted mandibular molar that has parallel mesial and

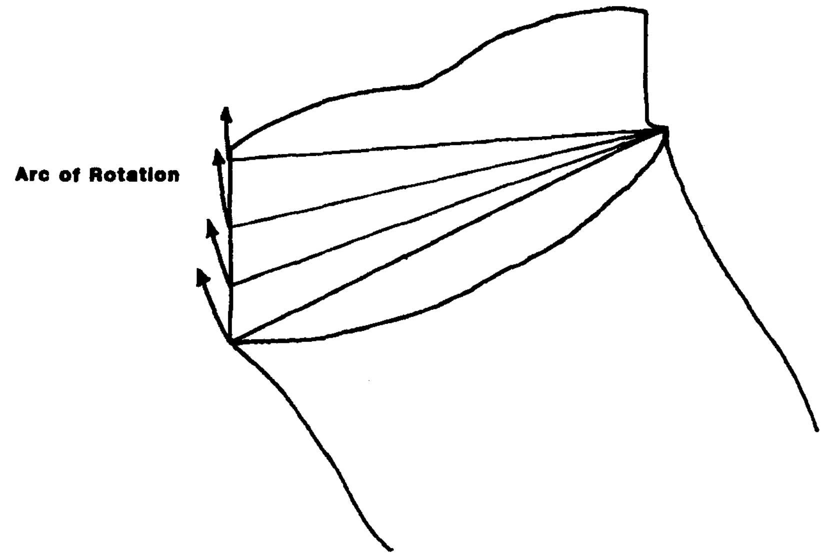

Fig. 8. Preparation of tilted mandibular molar showing that even though taper of mesial and distal walls is zero degrees, there is no resistance to distal rotation.

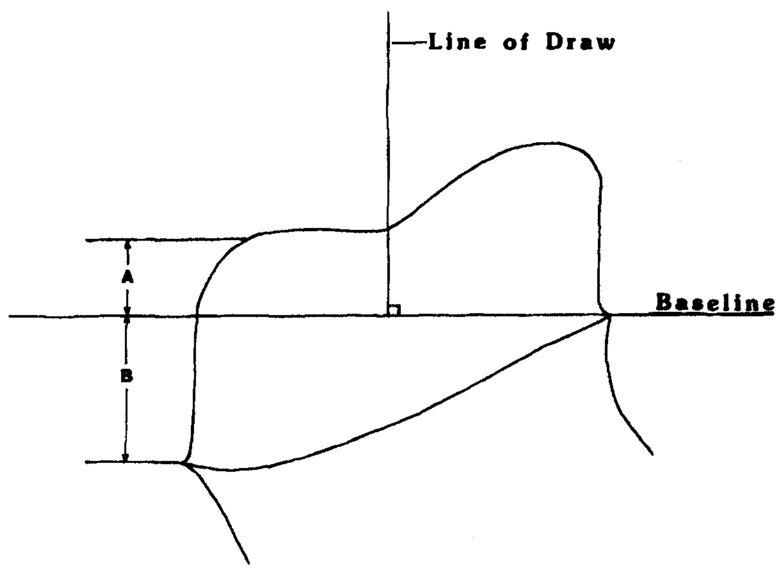

Fig. 9. For region *B* of this prepared tilted mandibular molar there is no feasible taper capable of providing resistance because all points in this region have a negative limiting taper. Only portion of mesial wall to be considered for evaluation of potential resistance is region *A*.

distal surfaces. The taper is zero, and the walls are of reasonable length. The methods of analysis of Weed and Baez[7] in their Fig. 5 with 0 degree convergence, and Gilboe and Teteruck[1] with adequate length, surface area, and perfect parallelism might indicate that this preparation would be resistive. However, by establishing the baseline through the distal center of rotation we see that the limiting taper of all points on the mesial wall are negative. Any point with a negative limiting taper cannot have resistance form. Thus the preparation does not have resistance to distal rotation of the restoration. This has been demonstrated by two- and three-dimensional models. If it is possible to have a model preparation with a "perfect" convergence of 0 degrees and not have resistance, the clinician must be wary under clinical conditions. It is not sufficient to just evaluate the length of the side, because all points on the mesial surface that are below the level of the distal margin are not resistive. The length to evaluate for resistance is only that portion above the level of the baseline (Fig. 9), and it is possible to lose all useful resistive length because of the tilt of the tooth. So for tilted abutments and any time that margins are at different levels, either mesial to distal or buccal to lingual, consideration of auxiliary resistance such as grooves is essential.

Analysis of groove placement

In considering the best location for groove placement, the objective is to make the limiting taper as large as possible, or from equation 2 to make h/b as large as possible. To achieve this, h must be as large and b as small as possible. The optimal location for groove placement is more coronal on the preparation and closer to the center of rotation. If resistance to rotation is lacking in both directions, two grooves on the same surface may be indicated, each near a center of rotation. For groove placement, a region with a negative limiting taper can provide resistance because although the direction of arc is away from one wall of the groove it would intersect the other wall. Portions of grooves at the level of the baseline where limiting taper is zero are least effective because the arc and the groove are in the same direction.

DISCUSSION

Even though 2 to 5 degrees has been taught for years as a standard for preparation taper and is a foundation principle of fixed prosthodontics,[9] it is not consistent with reality. Owen[10] states, "Most teeth are prepared with tapers in excess of 12 degrees and still function adequately. It is not known what retentive figure is the minimum required clinically." An explanation for the inability of retention to provide a minimal standard is that it is a continuous function of taper.[5] There is no demarcation to separate acceptable taper from unacceptable taper. The most that can be deduced is that less taper gives better retention.

Determination of adequate taper is more logically based on principles of resistance. Resistance form is a discontinous function of taper, having an *on* region and an *off* region. The taper separating the on region from the off region provides the standard of minimal acceptability. Resistance form is also said to be a more general principle and if a preparation has resistance form it will have retention, whereas the reverse is not true.[6]

Determination of adequate taper based on principles of resistance instead of retention was initiated by Weed and Baez.[7] They measured resistive forces for 10-, 13-, 16-, 19-, and 22-degree castings on a preparation 3.5 mm high and 10 mm at the base. The limiting taper for this preparation is 0.5 arcsin (0.7) or 22.2135 . . . degrees; thus all of the tapers tested had resistance form. They were all in the *on* region of tapers that provided resistance form. This is consistent with their observation that a sudden drop of the force required to dislodge a casting was not observed. Indication of the on-off nature would be expected for tests of castings with tapers larger than 22.2135 . . ., the limiting taper.

To dislodge by rotation a casting that has resistance form, the casting or the preparation must be distorted. This deformation can be of either a plastic (permanent) or elastic (nonpermanent) nature. Resistive tapers closer to the limiting taper would require less distortion and tend to be in the elastic range. As tapers that have resistance form or are in the *on* region become smaller, greater distortion would occur and require greater force. This again was observed by Weed and Baez,[7] that in the resistive region less taper required greater dislodging force and the castings were deformed. However, our basic premise for clinical acceptability and a minimally acceptable standard is that the preparation taper must only provide resistance form, not that it must resist rotational forces that would produce distortion. Forces that distort castings exceed those normally produced in the oral environment and thus would require resistive tapers less (better) than those reasonably expected for a minimal standard. The requirement that the preparation provide resistance form ensures that forces are resisted not by cement alone but also by the geometry of the preparation. The restoration can withstand all forces less than those that would distort the casting. This is a reasonable minimal standard for clinical acceptability.

CONCLUSION

This article presents a mathematical relationship between taper and resistance form that can serve as a foundation for establishing a minimal standard of acceptability for preparation taper more reasonably and clinically realistic than the usual recommended 3 to 5 degrees based on principles of retention. The concept of a limiting taper at each point is introduced as a number that divides the resistive from the nonresistive tapers. A preparation taper at a point less than the theoretically determined limiting taper is guaranteed to be resistive and tapers larger are not. This article presents a quantitative method of determining limiting taper for each point on a preparation as a function of location. It was demonstrated that for points on some preparations, 45 degrees is adequate to provide resistance form whereas for others even 0 degrees is not. This analysis is made point by point. Indication for the next study is to extend the analysis to the entire preparation, to determine an average taper value that guarantees that the entire preparation is resistive and therefore clinically sound. for the average dimensions of each tooth, molars to incisors, values can be calculated as guidelines for preparation taper.

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Accuracy of impression materials for complete-arch fixed partial dentures

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Dimensional accuracy of impression materials is crucial for the production of working casts in fixed prosthodontics. The accurate replication of tooth preparations and their arch position requires impression materials that exhibit limited distortion.

Research on dimensional stability of impression materials has commonly concentrated on the accuracy of individual dies[1-5] and quadrant fixed partial denture (FPD) abutments.[6-10] Studies reporting the accuracy of complete arch impressions have been sparse. Henry and Harnist[11] compared the accuracy of 14 different impression materials by using a four-posted, silver-plated, full-arch model and concluded that polyethers were the most reliable. In another study of full-arch impressions of two machined aluminum alloy dies, Lacy et al.[12] reported that the addition silicones were the most stable, followed by the polyethers. Stauffer et al.[13] studied the accuracy of six impression materials used in complete-arch FPD impressions with four intricate abutment preparations machined of stainless steel. A prosthesis was made to fit the master cast and the adaptation was evaluated on casts obtained from the various impression materials. They concluded that polyethers and addition silicones produced the most accurate working casts.

Table I. Impression materials used in study

Type of material	Manufacturer
Polyether	
Polyjel	L.D. Caulk, Milford, Del.
Impregum	Premier, ESPE, West Germany
Vinyl polysiloxane	
Reprosil	L.D. Caulk
Mirror 3	Sybron/Kerr, Romulus, Mich.
Polysulfide	
Neo Plex	Columbus Dental, St. Louis, Mo.
Permlastic	Sybron/Kerr
Reversible hydrocolloid	
Surgident	Columbus Dental
Agarloid	Van R, Los Angeles, Calif.
Irreversible hydrocolloid	
Jeltrate Plus	L.D. Caulk
Shurgel	Columbus Dental
Irreversible-reversible hydrocolloid combination	
Dentloid and Jeltrate Plus	Denterials, St. Louis, Mo. L.D. Caulk
Dentloid and Jeltrate	Denterials L.D. Caulk

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